Reference Model Based Learning in Expectation Formation: Experimental Evidence

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What?

How do people form expectations about future prices in financial market?

REH

- Assume perfect rationality
  - perfect knowledge for underlying market equilibrium,
  - perfect knowledge about the beliefs of all other agents in the market,
  - mental capacity to calculate the REE,
  - ...

Simple Heuristics

- ADA, that including only most recent prediction (*) and realised price :

$$p_t^* = p_{t-1}^* + \bar{G}(p_{t-1} - p_{t-1}^*), \quad 0 < \bar{G} < 1 \tag{1}$$

- People adjust their predictions by adapting to the most recent prediction error at a *constant* weight
- **Problem**: implies subjects are very hard-working
  - People will perpetually adapt to the past prediction error until they reach 0 prediction error
- No, b/c Bao et al. (2022) suggests subjects **satisfice**
  - "If it ain't broke, don't fix it."
    - Best prediction = least square learning = minimize sum of squared prediction error
    - In the structural estimation $p = \alpha + \beta \times \text{weather} + \varepsilon$, where they are tasked to estimate α and β and be paid according to the prediction error of p
    - Subjects stop update α and β once the prediction error is small enough (not 0) for them

Reference-model based learning (RMBL): extends and generalizes ADA in two aspects

- **Modification 1**: *Dynamic* weighted average of the previous prediction and the last observed price,

$$p_t^* = p_{t-1}^* + G_t(p_{t-1} - p_{t-1}^*) \tag{2}$$

- ΔG similar as in Hommes and Sorger (1998), but assume an myopic agent.
  - For $e_t = p_t - p_{t-1}$:
    - $\uparrow G$ if $cov(e_t \times e_{t-1}) > 0$: under-prediction followed by under-prediction, adaption was too timid, increase the adaption coefficient G
    - $\downarrow G$ if $cov(e_t \times e_{t-1}) < 0$: under-prediction followed by over-prediction, adaption was too aggressive, decrease the adaption coefficient G
  - Hommes and Sorger (1998): adapt to LR price $\bar{p}$ instead of p_{t-1}^*, and consider the sample autocorrelation of full history prediction error

RMBL extends and generalizes ADA in two aspects, contd.

- **Modification 2**: Implement a Stopping / Satisficing Rule
  - Adjust adaption coefficient G = speed up learning
  - Only speed up learning when the prediction error e_t is larger than an *objective* threshold Z
  - i.e., $|\Delta G| > 0$ only if $(e_t)^2 - Z > 0$
- A reference model can be *any* model, so that Z can be any Z
  - reference model defined as Kalman filter (i.e., ADA) in Bossaerts (2018) but "desired level of mean return and return volatility" when it is later applied to asset pricing in Berrada et al. (2024)
- Assume Z_i: maximum allowable threshold for each subject
- $Z_i > 0$: satisficing

RMBL extends and generalizes ADA in two aspects: Summary

$$p_t^* = p_{t-1}^* + G_t(p_{t-1} - p_{t-1}^*)$$

- ADA(Evans and Honkapohja, 2001): G_t is a constant
- Incremental Delta-Bar-Delta Algorithm (IDBD); d'Acremont and Bossaerts, 2016): $\Delta G_{t+1,t} > 0$ is a function of $Cov(e_t, e_{t-1})$
  - the process never end until $e_t = 0$, i.e., never satisfice
- RMBL(Bossaerts, 2018): $\Delta G_{t+1,t} > 0$ when $\Omega_t = e_t^2 - Z > 0$

How?

A horse race test to determine whether the expectation formation fits more closely to

- RMBL
- IDBD (RMBL without satisficing)
- ADA (IDBD with a constant gain factor),

using data from Learning to Forecast Experiments (LtFEs).

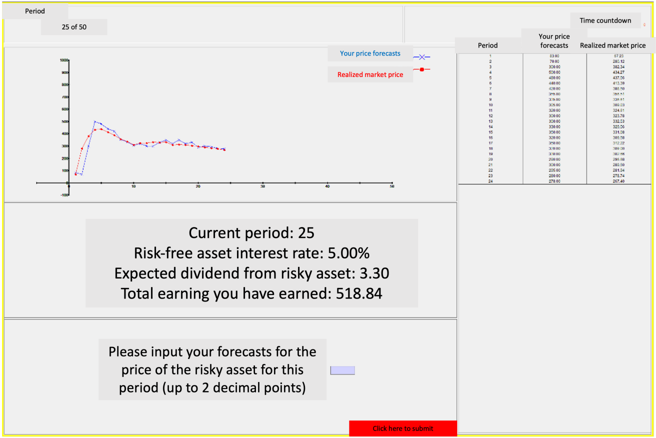
Dataset and why LtFEs

Expectation formation data from five set LtFEs (Bao et al., 2012; 2013; 2017; Bao and Hommes, 2019; Bao et al., 2024)

- Rich observations of 41,490 predictions from 801 subjects
- Allow full history of realized prices and predictions.
- Incentivize subjects to submit the most accurate prediction rather than to strategize.
  - avoid "testing join hypothesis" problem in traditional markets where ppl subject quantity decisions

summary on dataset

Typical Interface of a LtFE



- 6-10 subject in each market, 40-65 consecutive periods
- subject play the role of professional forecasters, payoff function is a inverse function with prediction error
- no knowledge on DGP (e.g., $p(t) = \frac{1}{1+r}(\bar{p}^e(t) + d) + e_t)$: know d, r but not $\bar{p}^e(t)$ = play with the market

Strategy

We do not have information on maximum allowable error Z_i in existing LtFEs.

Instead, we implement remedy of:

- Continuous analysis: when error is larger, ...
- Discrete analysis: when error is larger-than-individual-median, ...

do subject more likely to increase (decrease) G when the most two recent errors are positively (negatively) correlated?

Continuous analyses

RMBL: whether estimated continuous learning rate — possibility that one increases G when most recent two errors are positively correlated — is higher, when error is higher.

Subject FE logit:

$$Y_{i,t+1,t} = \sum_{j=1}^{N} D_{ij}\alpha_i^c + \beta^c E_{i,t} + \gamma^c R_{i,t,t-1} + \delta^c (E_{i,t} \times R_{i,t,t-1}) + \epsilon_{i,t} \quad (3)$$

- $Y_{i,t}$ (increase G, binary): it equals to 1 if at period t, subject i increases G in period $t+1$, in other words $\Delta G_{i,t+1,t} > 0$; and equals 0 if $\Delta G_{i,t+1,t} < 0$
- $R_{i,t,t-1}$ (Positively correlated Error, binary): equals to 1 if $Cov(e_{i,t}, e_{i,t-1}) = e_{i,t}e_{i,t-1} > 0$; equals to 0 if $Cov(e_{i,t}, e_{i,t-1}) < 0$.
- $E_{i,t}$ denotes the absolute prediction error subject i incurs at period t, i.e., $|e_{i,t}|$, where $e_{i,t} = p_{i,t} - p^*_{i,t}$.
  - instead of squared error, b/c squared error can be very large (e.g., up to a maximum of 648,073 in Model 6), making the coefficient hard to interpret

Continuous analyses: Hypothesis

$$Y_{i,t+1,t} = \sum_{j=1}^{N} D_{ij}\alpha_i^c + \beta^c E_{i,t} + \gamma^c R_{i,t,t-1} + \delta^c (E_{i,t} \times R_{i,t,t-1}) + \epsilon_{i,t} \tag{4}$$

- **RMBL:**
  - $\delta^c > 0$: when error $E_{i,t}$ is larger, subjects are more likely to increase G when the most recent two errors are positively correlated.
  - $\gamma^c \geq 0$: non-negative correlation between $R_{i,t,t-1}$ and $Y_{i,t}$
    - Zero correlation: allowing for $\Delta G_{i,t+1,t} = 0$ when error is large
- **IDBD:**
  - $\delta^c = 0$; $\gamma^c > 0$
- **ADA:**
  - $\delta^c = 0$; $\gamma^c = 0$

Discrete analyses: Hypothesis

Define $SE_{i,t} = 1$ if error is larger than individual median

- Median (instead of average) for balanced sample size & remove effect from outlier.
- Smaller percentile is more intuitive but favor RMBL in split-sample comparison.

$$Y_{i,t+1,t} = \sum_{j=1}^{N} D_{ij}\alpha_i^d + \beta^d \mathrm{SE}_{i,t} + \gamma^d R_{i,t,t-1} + \delta^d(\mathrm{SE}_{i,t} \times R_{i,t,t-1}) + \epsilon_{i,t} \tag{5}$$

Discrete analyses: Hypothesis, contd.

$$Y_{i,t+1,t} = \sum_{j=1}^{N} D_{ij}\alpha_i^d + \beta^d \mathrm{SE}_{i,t} + \gamma^d R_{i,t,t-1} + \delta^d(\mathrm{SE}_{i,t} \times R_{i,t,t-1}) + \epsilon_{i,t}$$

RMBL:

larger-than-median error:

$$\frac{dY_{i,t+1,t}}{dR_{i,t,t-1}} = \gamma^d + \delta^d\, SE_{i,t} > 0 \quad \text{when } SE_{i,t} = 0 \tag{6}$$

smaller-than-median error:

$$\frac{dY_{i,t+1,t}}{dR_{i,t,t-1}} = \gamma^d + \delta^d\, SE_{i,t} \geq 0 \quad \text{when } SE_{i,t} = 1 \tag{7}$$

- Equation (6): $\delta^d < 0$; $\gamma^d > 0$
- Equation (7): location of Z_i
  - When Z_i is at the median of $E_{i,t}$, $\frac{dY_{i,t+1,t}}{dR_{i,t,t-1}} = 0$ at $SE_{i,t} = 1$, so that $\gamma^d + \delta^d = 0$
  - When Z_i is much smaller than the median of $E_{i,t}$, $\frac{dY_{i,t+1,t}}{dR_{i,t,t-1}} > 0$ at $SE_{i,t} = 1$, so that $\gamma^d + \delta^d > 0$

Discrete analyses: Hypothesis, contd.

$$Y_{i,t+1,t} = \sum_{j=1}^{N} D_{ij}\alpha_i^d + \beta^d \mathrm{SE}_{i,t} + \gamma^d R_{i,t,t-1} + \delta^d (\mathrm{SE}_{i,t} \times R_{i,t,t-1}) + \epsilon_{i,t}$$

IDBD:

- $\delta^d = 0$; $\gamma^d > 0$

ADA:

- $\delta^d = 0$; $\gamma^d = 0$

Findings

Finding 1: RMBL dominates. All the experiments in our dataset show signs of satisficing in at least one of the discrete or continuous analyses. Meanwhile, IDBD could also provide explanatory power for 3 out of the total of 18 experiments. pooled results

Finding 2: The observation where RMBL explains well on the learning behaviour in all the experiments are robust in split-sample comparison and cross-study analysis. split-sample results

Finding 3: Z is not a universal constant and display heterogeneity across the experiment. pooled results

estimated continuous learning speed

Findings, contd.

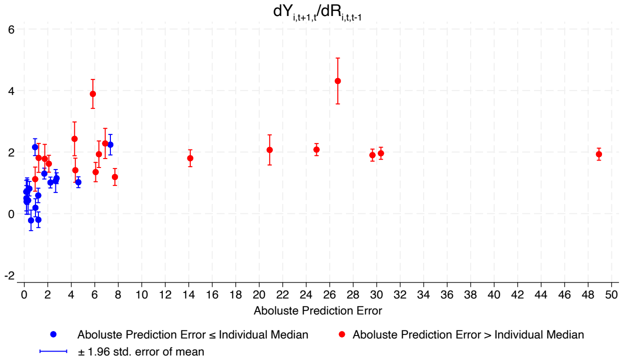


Figure 1: Coefficients of $\frac{dY_{(i,t+1,t)}}{dR_{(i,t,t-1)}}$ with regards to absolute prediction error in 18 experiments, separated by its absolute prediction error with regards to individual median.

Findings, contd.

For those models where Z_i is at the median of prediction error:

Table 1: Comparison of Median E_i in Models where $\gamma^d + \delta^d = 0$

	Median E_i (1)
Model 16 (LtFE) = 1	0.67***
	(0.06)
Model 17 (LtFE + LtOE Both) = 1	2.06***
	(0.15)
Model 18 (LtFE + LtOE Either) = 1	1.50***
	(0.08)
Constant (Model 14 (REE = 41, $21 \leq t \leq 43$) = 0)	0.51***
	(0.03)
Observations	150
R-squared	0.70

Note: Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Conclusion and Contributions

- First economic experiment with large sample size (observation = 41,490; #Subject = 801) to test RMBL.
  - RMBL that people adjust how they adapt to past prediction errors with regards to the correlation of the error term;
  - Meanwhile, they exhibit satisficing behavior, where they would only do so when the most recent prediction error is larger than their maximum allowable threshold.
- We find evidence that RMBL, the generalized ADA, explains the data in LtFEs (regardless of its feedback system) well.
  - Consistent with the satisficing evidence in LtFE when subjects are tasked to provide structural estimation (Bao et al., 2022)
  - Consistent with evidence found in neuroscience study using fMRI (d'Acremont and Bossaerts, 2016; N=21; Danckert et al., 2012; N=35)

Limitations and Future Research

- Causal study
  - Manipulating Z: tell subjects the median/average payoff per period (e.g., in a certain game)
  - Directing asking Z
    - Post-experiment questionnaire
    - Algo-trading experiment
- Alternative explanation on Z:
  - Instead of a maximum allowable error, it is a minimum allowable profit

Thank you !

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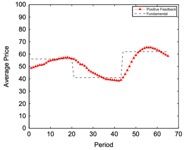
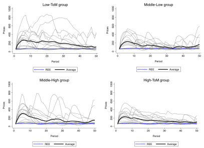
Appendix

Summary of Dataset

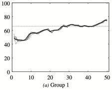
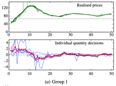
Table A.1: Summary of the Dataset Used: Positive Feedback Market

<table>
<tr><th>Study / Abbrev</th><th>Description</th><th>Treatment</th><th>Summary Statistics</th><th>Model</th><th>Realized Price Dynamics</th></tr>
<tr><td rowspan="3">Bao et al. (2012), JEDC / LtFE in Positive and Negative Feedback Market</td><td rowspan="3">LtFEs investigate the converge behaviour in positive and feedback market. They find that negative feedback market converge quickly while positive feedback market do not and show underreaction to short run and overreaction in the long run.
- Market size = 6
- # Subject = 48 in each treatment
- Convergence to REE: ×
- Within-subject design, from (1) to (2) to (3)</td><td>REE = 56, $1 \leq t \leq 20$</td><td>Var(Price): 14.6
E(|PE|): 0.973
#Obs: 960</td><td>(1)</td><td rowspan="3">
</td></tr>
<tr><td>REE = 41, $21 \leq t \leq 43$</td><td>Var(Price): 47.7
E(|PE|): 0.573
#Obs: 1104</td><td>(2)</td></tr>
<tr><td>REE = 62, $44 \leq t \leq 65$</td><td>Var(Price): 67.1
E(|PE|): 0.744
#Obs: 1056</td><td>(3)</td></tr>
<tr><td rowspan="4">Bao et al.(2024), JEBO / Theory of Mind (ToM)</td><td rowspan="4">LtFEs investigate whether market become more stable, resulting in lower volatility and fewer price bubbles when it is filled with people high theory of mind (ToM) capability, compared with the counterpart that filled with low ToM subjects. No significant differences are found.
- Market size = 6
- # Subject = 96 in each treatment
- # Obs = 4800 in each treatment
- Convergence to REE: ×
- Between- subject</td><td>High ToM</td><td>Var(Price): 9347.0
E(|PE|): 13.56</td><td>(4)</td><td rowspan="4">
</td></tr>
<tr><td>Medium High</td><td>Var(Price): 21963.2
E(|PE|): 16.25</td><td>(5)</td></tr>
<tr><td>Medium Low</td><td>Var(Price): 10444.8
E(|PE|): 16.04</td><td>(6)</td></tr>
<tr><td>Low ToM</td><td>Var(Price): 33306.2
E(|PE|): 26.80</td><td>(7)</td></tr>
</table>

Note: In the column of realized price dynamics, y-axis denotes the average price while x-axis represents the period. There are 70 periods in Bao et al. (2012) while only 50 periods in Bao et al. (2024). In both studies, the dotted line are the fundamental value or rational expectation equilibrium (REE) of the price, while the solid lines are the realized market price (which is a function of all subjects prediction on the price). As the solid line is still far away from the dotted line, it is concluded that the price does not converge to REE at the end of the experiment. The quantitative approach of measuring whether the price converges using relative and absolute deviation can be found in respective original studies. PE stands for prediction error, i.e., PE = $p_t - p_t^*$.

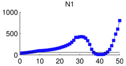
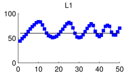
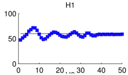
Appendix: Dataset

Table A.2: Summary of the Dataset Used: Positive Feedback Market

Study / Abbrev	**Description**	**Treatment**	**Summary Statistics**	**Model**	**Realized Price Dynamics**
Bao et al. (2017), EJ / LtFE vs. LtOE Positive	Compare the price dynamics and bubbles formation in asset across three treatments: (1) LtFE where subjects submit price only; (2) LtOE where subjects choose quantity to buy/sell; (3) perform both tasks, where payoff depends on price or quantity decision in equal probability. They find that bubble is larger in (2) and (3) compared to (1). - Exclude data in (2) because no price prediction - Market size = 6 - # Subject = 48 in each treatment - # Obs = 2400 in each treatment - Convergence to REE: × - Between- subject	LtFE LtFE + LtOE Both	Var(Price): 71.3 E(|PE|): 1.267 Var(Price): 1416.8 E(|PE|): 7.665	(8) (9)	  *Left: LtFE in (1); Right: Mixed in (3)*

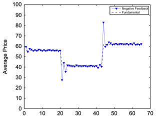
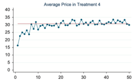
Note: In the column of realized price dynamics, y-axis denotes the average price while x-axis represents the period. There are 50 periods of the game. The dotted black lines are the fundamental value or rational expectation equilibrium (REE) of the price or the quantity, while the solid lines are the realized market price or quantity (which is a function of all subjects prediction/decision on the price). As the solid line of price prediction in both graph are still far away from the dotted line, it is concluded that the price does not converge to REE at the end of the experiment. The quantitative approach of measuring whether the price converges using relative and absolute deviation can be found in respective original studies. PE stands for prediction error, i.e., PE = $p_t - p_t^*$.

Table A.3: Summary of the Dataset Used: Positive Feedback Market

Study / Abbrev	Description	Treatment	Summary Statistics	Model	Realized Price Dynamics
Bao and Hommes (2019), JEDC / Speculator vs. Supplier in Housing Market	Housing market is a combination of positive feedback market (through speculative demand) and negative feedback market (through endogenous supply of housing). The study designs an experimental housing market and study how the strength of the negative feedback, the price elasticity of supply (PES), affect market stability. The result suggests that market stabilizes and price converge to REE only when there is strong PES where there is elastic housing supply (Treatment H: PES = 0.25), but fail to do so when there is no supplier (Treatment N: PES = 0) or when PES is low (Treatment L: PES = 0.1). - Market size = 6 in N, Market size = 9 in L and H - # Subject: Treatment N = 24; Treatment L = 45; Treatment H = 54 - # Obs: Treatment N = 1200; Treatment L = 2250; Treatment H= 2700 - Between-subject	No Supplier (N)	Var(Price): 11004.0 E(|PE|): 11.78 Converge to REE? ×	(10)	
		Low PES (L)	Var(Price): 265.0 E(|PE|): 17.01 Converge to REE? ×	(11)	
		High PES (H)	Var(Price): 24.0 E(|PE|): 3.386 Converge to REE? ✓	(12)	

Note: In the column of realized price dynamics, y-axis denotes the average price while x-axis represents the period. There are 50 periods in total. The black line are the fundamental value or rational expectation equilibrium (REE) of the price, while the blue line is the realized market price (which is a function of all subjects prediction on the price). As the solid line in N1 and H1 is still far away from the black line at the end of the experiment, while stick around the black line in H1, we conclude that only H1 converge to REE. The quantitative approach of measuring whether the price converges using relative and absolute deviation can be found in respective original studies. PE stands for prediction error, i.e., PE = $p_t - p_t^*$.

Table A.4: Summary of the Dataset Used: Negative Feedback Market

Study / Abbrev	Description	Treatment	Summary Statistics	Model	Realized Price Dynamics
Bao et al. (2012), JEDC / LtFE in Positive and Negative Feedback Market	*Same as in Model 1 – 3:* LtFEs investigate the converge behaviour in positive and feedback market. They find that negative feedback market converge quickly while positive feedback market do not and show underreaction to short run and overreaction in the long run. - Market size = 6 - # Subject = 48 in each treatment - Convergence to REE: ✓ Within-subject design, from (1) to (2) to (3)	REE = 56, $1 \leq t \leq 20$	Var(Price): 3.5 E(|PE|): 2.314 #Obs: 960	(13)	
		REE = 41, $21 \leq t \leq 43$	Var(Price): 11.7 E(|PE|): 3.426 #Obs: 1104	(14)	
		REE = 62, $44 \leq t \leq 65$	Var(Price): 21.9 E(|PE|): 3.591 #Obs: 1056	(15)	
Bao et al. (2013), EER / LtFE vs. LtOE Negative	Consider both forecasting (LtFE) and optimization decisions (LtOE) in a negative feedback market (i.e., experimental cobweb economy). The treatment include (1) LtFE: price forecasts only; (2) LtOE: quantity only; (3) LtFE + LtOE Both; (4) LtFE + LtOE Either, where they are paired in teams of 2, where one assigned with LtFE and another assigned with LtOE. All treatments converge to REE but at different speed. Performance is the best in (1) and worst in (3). - Exclude data in (2) because no price prediction - Market Size (i.e., number of subjects subject price prediction) in each treatment = 6 - # Valid Subject: LtFE: 24; LtFE+LtOE Both:42; LtFE + LtOE Either: 36 - # Obs: LtFE: 1200; LtFE+LtOE Both:2100; LtFE + LtOE Either: 1800 - Convergence to REE: ✓ - Between- subject	LtFE	Var(Price): 5.7 E(|PE|): 2.465	(16)	  
		LtFE + LtOE Both	Var(Price): 56.7 E(|PE|): 4.463	(17)	
		LtFE + LtOE Either	Var(Price): 21.4 E(|PE|): 3.517	(18)	

Note: In the column of realized price dynamics, y-axis denotes the average price while x-axis represents the period. There are 50 periods in total. The smooth lines are the fundamental value or rational expectation equilibrium (REE) of the price, while the dotted line is the realized market price (which is a function of all subjects prediction on the price). As the smooth line is close to the dotted line in all the market, we conclude that price converge to REE. The quantitative approach of measuring whether the price converges using relative and absolute deviation can be found in respective original studies. PE stands for prediction error, i.e., PE = $p_t - p_t^*$.

dataset

Main Results: Pooled

Table A.5: $\frac{dY_{i,t+1,t}}{dR_{i,t,t-1}}$ in Positive Feedback Market

Study and Description	**LtFE in Positive and Negative Feedback Market / Bao et al. (2012), JEDC, *Positive Feedback Markets***			**Theory of Mind (ToM) / Bao et al (2024), JEBO**				**LtFE vs. LtOE Positive / Bao et al. (2017), EJ**		**Speculator vs. Supplier in Housing Market / Bao and Hommes (2019), JEDC**		
Treatment	REE = 56, $1 \le t \le 20$	REE = 41, $21 \le t \le 43$	REE = 62, $44 \le t \le 65$	High-ToM	Medium High	Medium Low	Low-ToM	LtFE	LtFE+LtOE Both	No Supplier	Low PES	High PES
Model	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: Continuous Analysis												
Positively Correlated PE × |PE|, δ^c	1.42***	0.76***	0.93***	0.02***	0.01***	0.01***	0.01***	0.55***	0.15***	0.04***	0.16***	0.33***
	(0.38)	(0.25)	(0.26)	(0.00)	(0.00)	(0.00)	(0.00)	(0.11)	(0.03)	(0.01)	(0.03)	(0.06)
Positively Correlated PE, γ^c	0.40*	0.55***	0.65***	1.34***	1.37***	1.47***	1.30***	0.62***	0.72***	1.09***	1.25***	2.05***
	(0.21)	(0.17)	(0.20)	(0.07)	(0.07)	(0.07)	(0.07)	(0.14)	(0.12)	(0.18)	(0.22)	(0.15)
|PE|, β^c	-0.89***	-0.24*	-0.58***	-0.01***	-0.00**	-0.00***	-0.00***	-0.27***	-0.14***	-0.01	-0.12***	-0.14**
	(0.33)	(0.14)	(0.21)	(0.00)	(0.00)	(0.00)	(0.00)	(0.08)	(0.03)	(0.01)	(0.03)	(0.06)
Observations	852	1,053	978	4,558	4,576	4,572	4,551	2,246	2,269	1,138	2,142	2,513
Number of Subject	48	48	48	96	96	96	96	48	48	24	45	54
Classification	**RMBL**	**RMBL**	**RMBL**	**RMBL**	**RMBL**	**RMBL**	**RMBL**	**RMBL**	**RMBL**	**RMBL**	**RMBL**	**RMBL**
Panel B: Discrete Analysis												
Positively Correlated PE × Small |PE|, δ^d	-1.21***	-0.39	-1.10***	-1.11***	-0.78***	-0.66***	-0.91***	-0.75***	-1.24***	-1.04***	-1.92***	-1.77***
	(0.30)	(0.26)	(0.29)	(0.13)	(0.13)	(0.13)	(0.13)	(0.18)	(0.18)	(0.30)	(0.39)	(0.27)
Positively Correlated PE, γ^d	1.70***	1.08***	1.80***	2.10***	1.93***	1.95***	1.96***	1.59***	1.81***	2.09***	4.16***	3.92***
	(0.23)	(0.19)	(0.22)	(0.10)	(0.10)	(0.10)	(0.10)	(0.13)	(0.14)	(0.24)	(0.36)	(0.23)
Small |PE|, β^d	0.61***	-0.16	0.46**	0.52***	0.25**	0.28***	0.38***	0.21	0.70***	0.61**	1.18***	0.84***
	(0.22)	(0.20)	(0.22)	(0.11)	(0.10)	(0.10)	(0.10)	(0.13)	(0.13)	(0.26)	(0.38)	(0.24)
Observations	852	1,053	978	4,558	4,576	4,572	4,551	2,246	2,269	1,138	2,142	2,513
Number of Subject	48	48	48	96	96	96	96	48	48	24	45	54
Classification	**RMBL**	**IDBD**	**RMBL**	**RMBL**	**RMBL**	**RMBL**	**RMBL**	**RMBL**	**RMBL**	**RMBL**	**RMBL**	**RMBL**
Test: $\gamma^d + \delta^d$	0.491**		0.692***	0.989***	1.145***	1.295***	1.045***	0.840***	0.568***	1.054***	2.240***	2.144***
	(0.2)		(0.19)	(0.09)	(0.09)	(0.09)	(0.09)	(0.12)	(0.12)	(0.19)	(0.17)	(0.14)
E (Median of $\mathcal{E}_i$)	0.391		0.522	5.168	6.201	3.771	12.11	0.919	2.432	5.905	13.90	2.142

Note: Logit estimates fit for panel data with subject level fixed effect (except for Model 9 in Panel A where subject level fixed effect model cannot converge, so that a random effect model is implemented). PE stands for prediction error, i.e., PE = $p_t - p_t^*$. *** p<0.01, ** p<0.05, * p<0.1.

Table A.6: $\frac{dY_{i,t+1,t}}{dR_{i,t,t-1}}$ in Negative Feedback Market

Study and Description	LtFE in Positive and Negative Feedback Market / Bao et al. (2012), JEDC, *Negative Feedback Markets*			LtFE vs. LtOE Negative / Bao et al. (2013), EER		
Treatment	REE = 56, $1 \le t \le 20$	REE = 41, $21 \le t \le 43$	REE = 62, $44 \le t \le 65$	LtFE	LtFE+LtOE Both	LtFE+LtOE Either
Model	(13)	(14)	(15)	(16)	(17)	(18)
Panel A: Continuous Analysis						
Positively Correlated PE × |PE|, δ^c	0.38***	0.04*	0.02	0.47***	0.14***	0.20***
	(0.10)	(0.02)	(0.02)	(0.09)	(0.02)	(0.04)
Positively Correlated PE, γ^c	0.75***	1.10***	1.33***	-0.21	-0.05	0.20
	(0.20)	(0.16)	(0.17)	(0.17)	(0.13)	(0.14)
|PE|, β^c	-0.16***	0.01	-0.01	-0.12***	-0.04***	-0.05***
	(0.05)	(0.01)	(0.01)	(0.03)	(0.01)	(0.02)
Observations	791	918	826	1,087	1,846	1,537
Number of Subject	48	48	48	24	42	36
Classification	**RMBL**	**IDBD**	**IDBD**	**RMBL**	**RMBL**	**RMBL**
Panel B: Discrete Analysis						
Positively Correlated PE × Small |PE|, δ^d	-1.84***	-1.51***	-1.41***	-1.58***	-1.37***	-1.19***
	(0.32)	(0.29)	(0.31)	(0.26)	(0.19)	(0.21)
Positively Correlated PE, γ^d	2.28***	1.91***	2.11***	1.39***	1.16***	1.36***
	(0.25)	(0.21)	(0.22)	(0.19)	(0.14)	(0.15)
Small |PE|, β^d	1.10***	0.56***	0.51***	0.79***	0.64***	0.54***
	(0.20)	(0.19)	(0.20)	(0.16)	(0.13)	(0.14)
Observations	791	918	826	1,087	1,846	1,537
Number of Subject	48	48	48	24	42	36
Classification	**RMBL**	**RMBL**	**RMBL**	**RMBL**	**RMBL**	**RMBL**
Test: $\gamma^d + \delta^d$	0.433**	0.401*	0.694***	-0.196	-0.214	0.170
	(0.22)	(0.2)	(0.22)	(0.17)	(0.13)	(0.15)
E (Median of $\mathcal{E}_i$)	0.782	**0.508**	0.489	**1.182**	**2.568**	**2.006**

Note: Logit estimates fit for panel data with subject level fixed effect PE stands for prediction error, i.e., PE = $p_t - p_t^*$. *** p<0.01, ** p<0.05, * p<0.1.

finding

Main Results: Split Sample

Table A.7: $\frac{dY_{i,t+1,t}}{dR_{i,t,t-1}}$ in Positive Feedback Market: Split Sample

Study and Description	**LtFE in Positive and Negative Feedback Market / Bao et al. (2012), JEDC, *Positive Feedback Markets***			**Theory of Mind (ToM) / Bao et al (2024), JEBO**				**LtFE vs. LtOE Positive / Bao et al. (2017), EJ**		**Speculator vs. Supplier in Housing Market / Bao and Hommes (2019), JEDC**		
Treatment	REE = 56, $1 \leq t \leq 20$	REE = 41, $21 \leq t \leq 43$	REE = 62, $44 \leq t \leq 65$	High-ToM	Medium High ToM	Medium Low ToM	Low-ToM	LtFE	LtFE+LtOE Both	No Supplier	Low PES	High PES
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: Error Smaller than Subject-Level Median (Small Error = 1)												
Positively Correlated PE	0.50**	0.71***	0.71***	1.01***	1.15***	1.30***	1.02***	0.81***	0.59***	1.06***	2.24***	2.16***
	(0.21)	(0.19)	(0.20)	(0.09)	(0.09)	(0.09)	(0.09)	(0.12)	(0.12)	(0.19)	(0.17)	(0.14)
Observations	433	486	473	2,264	2,275	2,308	2,266	1,152	1,150	573	1,058	1,221
Number of Subject	48	48	47	96	96	96	96	48	48	24	45	54
Panel B: Error Larger than or Equal to Subject-Level Median (Small Error = 0)												
Positively Correlated PE	1.78***	1.12***	1.81***	2.08***	1.90***	1.96***	1.93***	1.62***	1.80***	2.07***	4.31***	3.89***
	(0.24)	(0.20)	(0.24)	(0.10)	(0.10)	(0.10)	(0.10)	(0.14)	(0.14)	(0.25)	(0.38)	(0.24)
Observations	419	567	501	2,294	2,301	2,264	2,285	1,094	1,119	565	1,084	1,292
Number of Subject	48	48	48	96	96	96	96	48	48	24	45	54

Note: Logit estimates fit for panel data with subject level fixed effect. PE stands for prediction error, i.e., PE = $p_t - p_t^*$. *** p<0.01, ** p<0.05, * p<0.1.

Table A.8: $\frac{dY_{i,t+1,t}}{dR_{i,t,t-1}}$ in Negative Feedback Market: Split Sample

Study and Description	**LtFE in Positive and Negative Feedback Market / Bao et al. (2012), JEDC, *Negative Feedback Markets***			**LtFE vs. LtOE Negative / Bao et al. (2013), EER**		
Treatment	REE = 56, $1 \leq t \leq 20$	REE = 41, $21 \leq t \leq 43$	REE = 62, $44 \leq t \leq 65$	LtFE	LtFE+LtOE Both	LtFE+LtOE Either
	(13)	(14)	(15)	(16)	(17)	(18)
Panel A: Error Smaller than Subject-Level Median (Small Error = 1)						
Positively Correlated PE	0.43*	0.39*	0.73***	-0.22	-0.20	0.19
	(0.23)	(0.21)	(0.22)	(0.17)	(0.13)	(0.15)
Observations	376	383	358	553	902	745
Number of Subject	43	41	41	24	42	36
Panel B: Error Larger than or Equal to Subject-Level Median (Small Error = 0)						
Positively Correlated PE	2.43***	1.93***	2.28***	1.41***	1.19***	1.35***
	(0.28)	(0.22)	(0.25)	(0.20)	(0.14)	(0.16)
Observations	401	532	457	534	944	792
Number of Subject	47	48	48	24	42	36

Note: Logit estimates fit for panel data with subject level fixed effect. PE stands for prediction error, i.e., PE = $p_t - p_t^*$. *** p<0.01, ** p<0.05, * p<0.1.

finding

Estimated Continuous Learning Speed

- **Estimated Continuous Learning Rate** — the **probability** that subject to increase (decrease) G when the error term in the most recent two periods are positively (negatively) correlated — increases when prediction error is larger
  - $\frac{dY_{i,t+1,t}}{dR_{i,t,t-1}} = +f(E_{i,t}) = -f(SE_{i,t})$
- **Estimated Continuous Learning Speed** — the **magnitude** of increment (decrement) in G when error term in the most recent periods are positively (negatively) correlated — increases when prediction error is larger
  - not predicted in ADA, RMBL, or IDBD
- Same testable hypothesis, but with different interpretation

$$\Delta G_{i,t+1,t} = \sum_{j=1}^{N} D_{ij}\alpha_i^c + \beta^c E_{i,t} + \gamma^c R_{i,t,t-1} + \delta^c (E_{i,t} \times R_{i,t,t-1}) + \epsilon_{i,t} \quad (8)$$

$$\Delta G_{i,t+1,t} = \sum_{j=1}^{N} D_{ij}\alpha_i^d + \beta^d \mathrm{SE}_{i,t} + \gamma^d R_{i,t,t-1} + \delta^d (\mathrm{SE}_{i,t} \times R_{i,t,t-1}) + \epsilon_{i,t} \quad (9)$$

Estimated Continuous Learning Speed: Result

Finding 4: The results on estimated binary estimated continuous learning speed in RMBL can be extended to estimated continuous estimated continuous learning speed.

When conducting analyses that are robust to outliers, we find evidence that the estimated continuous estimated continuous learning speed—the increment in the magnitude of adaptive response with regard to the positive correlation of the error term—also increases when there is a larger absolute prediction error.

Estimated Continuous Learning Speed: OLS Pooled

Table A.9: $\frac{d\Delta G_{i,t+1,t}}{dR_{i,t,t-1}}$ in Positive Feedback Market

Study and Description	LtFE in Positive and Negative Feedback Market / Bao et al. (2012), JEDC, *Positive Feedback Markets*			Theory of Mind (ToM) / Bao et al (2024), JEBO				LtFE vs. LtOE Positive / Bao et al. (2017), EJ		Speculator vs. Supplier in Housing Market / Bao and Hommes (2019), JEDC		
Treatment	REE = 56, $1 \leq t \leq 20$	REE = 41, $21 \leq t \leq 43$	REE = 62, $44 \leq t \leq 65$	High-ToM	Medium High	Medium Low	Low-ToM	LtFE	LtFE+LtOE Both	No Supplier	Low PES	High PES
Model	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: Continuous Analysis												
Positively Correlated PE × |PE|, δ^c	7.86**	3.25***	3.29	1.16**	0.15***	0.59***	0.17**	26.77**	0.66***	3.43***	-0.14	0.14
	(3.65)	(0.63)	(2.34)	(0.49)	(0.03)	(0.20)	(0.07)	(12.86)	(0.22)	(0.30)	(0.13)	(0.14)
Positively Correlated PE, γ^c	-0.68	2.93**	5.23	-2.80	10.71***	4.87	6.01**	-25.87*	8.60***	-11.62	8.69***	5.14***
	(1.85)	(1.37)	(3.65)	(6.80)	(2.30)	(3.99)	(2.67)	(13.94)	(3.08)	(10.26)	(2.64)	(0.74)
|PE|, β^c	-4.85	-1.41**	-1.85	-1.01**	-0.04*	-0.15*	-0.04	-26.53**	-0.32	0.22	0.10	-0.12
	(3.22)	(0.61)	(1.95)	(0.50)	(0.02)	(0.08)	(0.04)	(13.10)	(0.22)	(0.24)	(0.12)	(0.12)
Observations	894	1,077	994	4,598	4,590	4,594	4,590	2,283	2,302	1,152	2,160	2,586
R-squared	0.06	0.03	0.01	0.09	0.01	0.04	0.00	0.27	0.30	0.43	0.03	0.05
Number of Subject	48	48	48	96	96	96	96	48	48	24	45	54
Classification	RMBL	RMBL	ADA	RMBL	RMBL	RMBL	RMBL	RMBL	RMBL	RMBL	IDBD	IDBD
Panel B: Discrete Analysis												
Positively Correlated PE × Small |PE|, δ^d	-3.32*	0.07	4.30	-10.72	-4.08	-11.65	-10.63	-5.84	-4.27	9.90	2.36	0.03
	(1.75)	(2.08)	(6.06)	(9.25)	(3.46)	(8.64)	(14.69)	(6.03)	(3.69)	(7.61)	(1.82)	(1.07)
Positively Correlated PE, γ^d	5.16***	4.70**	5.12*	19.72***	15.17***	20.01***	15.97**	5.25	16.71***	21.47	5.42***	5.34***
	(1.13)	(1.81)	(2.56)	(5.86)	(2.43)	(4.18)	(7.44)	(3.36)	(5.33)	(13.36)	(0.69)	(0.53)
Small |PE|, β^d	2.17	-0.09	-2.85	9.83	2.16	5.33	8.67	5.10	6.74	-20.46	-2.09	-0.43
	(1.84)	(2.11)	(4.77)	(8.71)	(3.63)	(7.11)	(14.33)	(5.97)	(6.12)	(16.15)	(1.65)	(0.72)
Observations	894	1,077	994	4,598	4,590	4,594	4,590	2,283	2,302	1,152	2,160	2,586
R-squared	0.02	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.01	0.01	0.03	0.05
Number of Subject	48	48	48	96	96	96	96	48	48	24	45	54
Classification	IDBD	IDBD	ADA	IDBD	IDBD	IDBD	IDBD	ADA	IDBD	ADA	IDBD	IDBD

Note: Subject level fixed effects OLS model with cluster-robust standard error for panels nested within subject level. PE stands for prediction error, i.e., PE = $p_t - p_t^*$. *** p<0.01, ** p<0.05, * p<0.1.

Table A.10: $\frac{d\Delta G_{i,t+1,t}}{dR_{i,t,t-1}}$ in Positive Feedback Market

Study and Description	LtFE in Positive and Negative Feedback Market / Bao et al. (2012), JEDC, *Negative Feedback Markets*			LtOE vs. LtOE Negative / Bao et al. (2013), EER		
Treatment	REE = 56, $1 \leq t \leq 20$	REE = 41, $21 \leq t \leq 43$	REE = 62, $44 \leq t \leq 65$	LtFE	LtFE+LtOE Both	LtFE+LtOE Either
Model	(13)	(14)	(15)	(16)	(17)	(18)
Panel A: Continuous Analysis						
Positively Correlated PE × |PE|, δ^c	-0.01	9.00	0.10	1.34	0.15**	0.19
	(0.13)	(7.09)	(0.16)	(1.06)	(0.07)	(0.15)
Positively Correlated PE, γ^c	2.21***	-24.57	3.16**	0.18	0.18	0.27
	(0.79)	(21.01)	(1.26)	(2.78)	(0.44)	(0.63)
|PE|, β^c	-0.07*	-0.33	-0.14	0.07	-0.12	-0.06
	(0.04)	(0.22)	(0.16)	(0.23)	(0.10)	(0.04)
Observations	2,586	910	1,088	998	1,150	2,012
R-squared	0.05	0.02	0.06	0.02	0.01	0.00
Number of Subject	54	48	48	48	24	42
Classification	IDBD	ADA	IDBD	ADA	RMBL	ADA
Panel B: Discrete Analysis						
Positively Correlated PE × Small |PE|, δ^d	-0.92	-41.25	-1.45	-0.44	-1.42**	-1.85**
	(0.62)	(33.50)	(0.96)	(3.06)	(0.55)	(0.71)
Positively Correlated PE, γ^d	2.74***	23.05	3.95***	2.61***	1.49***	1.78***
	(0.63)	(18.25)	(1.12)	(0.73)	(0.50)	(0.42)
Small |PE|, β^d	2.10***	3.89	0.80	-1.24	1.39**	1.27**
	(0.71)	(2.44)	(1.12)	(2.96)	(0.57)	(0.56)
Observations	910	1,088	998	1,150	2,012	1,726
R-squared	0.04	0.01	0.02	0.00	0.00	0.01
Number of Subject	48	48	48	24	42	36
Classification	IDBD	ADA	IDBD	IDBD	RMBL	RMBL

Note: Subject level fixed effects OLS model with cluster-robust standard error for panels nested within subject level. PE stands for prediction error, i.e., PE = $p_t - p_t^*$. *** p<0.01, ** p<0.05, * p<0.1.

Estimated Continuous Learning Speed: OLS Split Sample

Table A.11: $\frac{d\Delta G_{i,t+1,t}}{dR_{i,t,t-1}}$ in Negative Feedback Market

Study and Description	LtFE in Positive and Negative Feedback Market / Bao et al. (2012), JEDC, *Positive Feedback Markets*			Theory of Mind (ToM) / Bao et al (2024), JEBO				LtFE vs. LtOE Positive / Bao et al. (2017), EJ		Speculator vs. Supplier in Housing Market / Bao and Hommes (2019), JEDC		
Treatment	REE = 56, $1 \leq t \leq 20$	REE = 41, $21 \leq t \leq 43$	REE = 62, $44 \leq t \leq 65$	High-ToM	Medium High ToM	Medium Low ToM	Low-ToM	LtFE	LtFE+LtOE Both	No Supplier	Low PES	High PES
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: Error Smaller than Subject-Level Median (Small Error = 1)												
Positively Correlated PE	1.83	4.46**	9.56*	10.18**	11.82***	9.49	8.45	0.33	10.70**	31.68	7.56***	5.40***
	(1.66)	(2.07)	(4.90)	(4.32)	(3.11)	(5.73)	(5.72)	(1.72)	(4.56)	(21.10)	(1.81)	(0.97)
Observations	459	502	490	2,290	2,281	2,318	2,275	1,174	1,173	583	1,069	1,283
R-squared	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.04
Number of Subject	48	48	48	96	96	96	96	48	48	24	45	54
Panel B: Error Larger than or Equal to Subject-Level Median (Small Error = 0)												
Positively Correlated PE	5.15***	4.57**	5.42***	17.90***	14.99***	17.95***	15.27**	5.46	17.45***	26.63	5.15***	5.22***
	(1.28)	(1.73)	(1.96)	(5.37)	(2.46)	(3.65)	(7.15)	(3.31)	(5.79)	(17.88)	(0.63)	(0.52)
Observations	435	575	504	2,308	2,309	2,276	2,315	1,109	1,129	569	1,091	1,303
R-squared	0.05	0.02	0.02	0.01	0.03	0.01	0.00	0.00	0.01	0.00	0.02	0.07
Number of Subject	435	575	504	2,308	2,309	2,276	2,315	1,109	1,129	569	1,091	1,303

Note: Subject level fixed effects OLS model with cluster-robust standard error for panels nested within subject level. PE stands for prediction error, i.e., PE = $p_t - p_t^*$. *** p<0.01, ** p<0.05, * p<0.1.

Table A.12: $\frac{d\Delta G_{i,t+1,t}}{dR_{i,t,t-1}}$ in Positive Feedback Market: Split Sample

Study and Description	**LtFE in Positive and Negative Feedback Market / Bao et al. (2012), JEDC, *Negative Feedback Markets***			**LtFE vs. LtOE Negative / Bao et al. (2013), EER**		
Treatment	REE = 56, $1 \leq t \leq 20$	REE = 41, $21 \leq t \leq 43$	REE = 62, $44 \leq t \leq 65$	LtFE	LtFE+LtOE Both	LtFE+LtOE Either
	(13)	(14)	(15)	(16)	(17)	(18)
Panel A: Error Smaller than Subject-Level Median (Small Error = 1)						
Positively Correlated PE	1.79**	-16.65	2.16**	1.95	0.14	0.01
	(0.83)	(14.24)	(0.92)	(2.78)	(0.62)	(0.63)
Observations	474	509	486	590	1,023	872
R-squared	0.02	0.00	0.01	0.00	0.00	0.00
Number of Subject	48	48	48	24	42	36
Panel B: Error Larger than or Equal to Subject-Level Median (Small Error = 0)						
Positively Correlated PE	2.75***	27.00	4.10***	2.41***	1.41***	1.83***
	(0.70)	(23.14)	(1.11)	(0.78)	(0.48)	(0.46)
Observations	436	579	512	560	989	854
R-squared	0.05	0.01	0.03	0.02	0.01	0.02
Number of Subject	48	48	48	24	42	36

Note: Subject level fixed effects OLS model with cluster-robust standard error for panels nested within subject level. PE stands for prediction error, i.e., PE = $p_t - p_t^*$. *** p<0.01, ** p<0.05, * p<0.1.

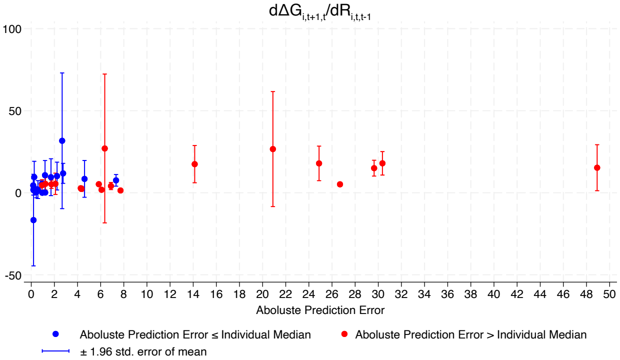


Figure A.1: Coefficients of $\frac{d\Delta G_{(i,t+1,t)}}{dR_{(i,t,t-1)}}$ with regards to absolute prediction error in 18 experiments, separated by its absolute prediction error with regards to individual median.

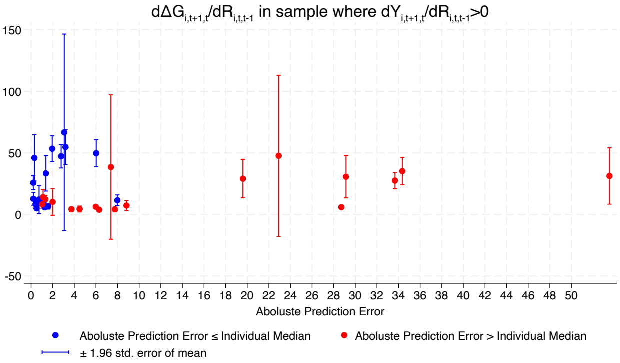


Figure A.2: Coefficients of $\frac{d\Delta G_{(i,t+1,t)}}{dR_{(i,t,t-1)}}$ in sample where $\frac{dY_{(i,t+1,t)}}{dR_{(i,t,t-1)}} > 0$ with regards to absolute prediction error in 18 experiments, separated by its absolute prediction error with regards to individual median.

Outliers from OLS!

- Increment of adaptive response with regard to a positively correlated error
- ... could be up to 30 units on average (with a standard error of 20 units) when the absolute prediction error is only 4 units.

Robust regression to outliers: M estimator (Huber, 1973)

- Different results from OLS
- When pooling all data, both continuous analysis ($\delta^c = 0.05$, $p < 0.01$) and discrete analysis ($\delta^d = -0.22$, $p < 0.05$) provide evidence supporting RMBL.
  - when the absolute prediction error is one unit larger, the increment in G with regard to the positively correlated error is 0.05 higher;
  - when the error is larger than the median, the increment in G with regard to the positively correlated error is 0.22 units higher — compared to if the error is smaller than the median.

- Splitting the sample according to the experiment
  - subjects in 15 out of the 18 experiments can be explained by the use of RMBL from at least one of the analyses.
- the evidence for RMBL (satisficing) is strong in all analyses, except for the discrete analysis in the positive-feedback market.
  - due to the limited variation in the explanatory variable in the discrete analysis.
  - X = 0 or 1
  - Y: much larger variability ($p < 0.01$) in the absolute ΔG in the positive feedback market ($\sigma(|\Delta G_{positive}|) = 94.20$) compared to that in the negative feedback market ($\sigma(|\Delta G_{negative}|) = 62.95$).

Estimated Continuous Learning Speed: M-estimator, Pooled

Table A.13: $\frac{d\Delta G_{i,t+1,t}}{dR_{i,t,t-1}}$ in Positive Feedback Market

Study and Description	**LtFE in Positive and Negative Feedback Market / Bao et al. (2012), JEDC, *Positive Feedback Markets***			**Theory of Mind (ToM) / Bao et al (2024), JEBO**				**LtFE vs. LtOE Positive / Bao et al. (2017), EJ**		**Speculator vs. Supplier in Housing Market / Bao and Hommes (2019), JEDC**		
Treatment	REE = 56, $1 \leq t \leq 20$	REE = 41, $21 \leq t \leq 43$	REE = 62, $44 \leq t \leq 65$	High-ToM	Medium High	Medium Low	Low-ToM	LtFE	LtFE+LtOE Both	No Supplier	Low PES	High PES
Model	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: Continuous Analysis												
Positively Correlated PE × |PE|, δ^c	2.20***	2.33***	4.48***	0.11***	0.08***	0.08***	0.02**	0.53*	0.52***	0.11	0.01	0.11***
	(0.40)	(0.64)	(1.18)	(0.03)	(0.02)	(0.02)	(0.01)	(0.27)	(0.02)	(0.06)	(0.02)	(0.03)
Positively Correlated PE, γ^c	0.51*	1.03**	1.22	2.93***	4.87***	4.61***	3.88***	0.52	0.21	2.84***	3.20***	3.27***
	(0.28)	(0.51)	(0.87)	(0.49)	(0.57)	(0.44)	(0.37)	(0.32)	(0.28)	(0.89)	(0.32)	(0.19)
|PE|, β^c	-2.07***	-0.78	-3.42***	-0.04**	-0.02	-0.02*	-0.00	-0.34*	-0.18***	-0.03***	-0.01	-0.11***
	(0.39)	(0.64)	(1.17)	(0.02)	(0.01)	(0.01)	(0.00)	(0.17)	(0.02)	(0.01)	(0.02)	(0.03)
Observations	894	1,077	994	4,598	4,590	4,594	4,590	2,283	2,302	1,152	2,160	2,586
Number of Subject	48	48	48	96	96	96	96	48	48	24	45	54
Classification	**RMBL**	**RMBL**	**RMBL**	**RMBL**	**RMBL**	**RMBL**	**RMBL**	**IDBD**	**RMBL**	**IDBD**	**IDBD**	**RMBL**
Panel B: Discrete Analysis												
Positively Correlated PE × Small |PE|, δ^d	-0.94*	2.18*	-1.84	0.18	0.75	1.02	0.62	0.12	-0.51	-1.28*	0.14	-0.30
	(0.49)	(1.22)	(1.51)	(0.70)	(0.90)	(0.83)	(0.62)	(0.19)	(0.39)	(0.75)	(0.32)	(0.24)
Positively Correlated PE, γ^d	2.03***	1.40**	5.29***	4.06***	5.77***	5.23***	4.14***	1.07***	2.15***	4.70***	3.17***	3.67***
	(0.31)	(0.57)	(0.83)	(0.36)	(0.46)	(0.39)	(0.31)	(0.11)	(0.24)	(0.61)	(0.25)	(0.18)
Small |PE|, β^d	0.92**	-1.74	2.25*	-0.22	-0.67	-0.65	-0.61	-0.13	0.85***	1.08	0.02	0.23
	(0.46)	(1.06)	(1.32)	(0.64)	(0.79)	(0.68)	(0.53)	(0.15)	(0.27)	(0.74)	(0.33)	(0.22)
Observations	894	1,077	994	4,598	4,590	4,594	4,590	2,283	2,302	1,152	2,160	2,586
R-squared	48	48	48	96	96	96	96	48	48	24	45	54
Classification	**IDBD**	**IDBD**	**IDBD**	**IDBD**	**IDBD**	**IDBD**	**IDBD**	**IDBD**	**IDBD**	**IDBD**	**IDBD**	**IDBD**

Note: Subject level fixed effects robust estimator fits for M regression models with cluster-robust standard error for panels nested within subject level. PE stands for prediction error, i.e., PE = $p_t - p_t^*$. *** p<0.01, ** p<0.05, * p<0.1.

Table A.14: $\frac{d\Delta G_{i,t+1,t}}{dR_{i,t,t-1}}$ in Negative Feedback Market

Study and Description	LtFE in Positive and Negative Feedback Market / Bao et al. (2012), JEDC, *Negative Feedback Markets*			LtFE vs. LtOE Negative / Bao et al. (2013), EER		
Treatment	REE = 56, $1 \le t \le 20$	REE = 41, $21 \le t \le 43$	REE = 62, $44 \le t \le 65$	LtFE	LtFE+LtOE Both	LtFE+LtOE Either
Model	(13)	(14)	(15)	(16)	(17)	(18)
Panel A: Continuous Analysis						
Positively Correlated PE × |PE|, δ^c	0.13***	0.01	0.01	0.23***	0.08***	0.09***
	(0.03)	(0.01)	(0.00)	(0.07)	(0.01)	(0.02)
Positively Correlated PE, γ^c	0.66***	0.57***	0.66***	-0.18	-0.05	0.18
	(0.15)	(0.15)	(0.14)	(0.19)	(0.11)	(0.12)
|PE|, β^c	-0.05***	0.00	-0.01*	-0.05***	-0.02***	-0.02***
	(0.01)	(0.01)	(0.00)	(0.01)	(0.01)	(0.01)
Observations	910	1,088	998	1,150	2,012	1,726
R-squared	48	48	48	24	42	36
Classification	**RMBL**	**IDBD**	**IDBD**	**RMBL**	**RMBL**	**RMBL**
Panel B: Discrete Analysis						
Positively Correlated PE × Small |PE|, δ^d	-0.82***	-0.48***	-0.64***	-0.95***	-0.79***	-0.61***
	(0.22)	(0.16)	(0.10)	(0.25)	(0.15)	(0.17)
Positively Correlated PE, γ^d	1.31***	0.82***	0.97***	0.72***	0.62***	0.74***
	(0.16)	(0.12)	(0.11)	(0.17)	(0.10)	(0.11)
Small |PE|, β^d	0.70***	0.22**	0.14*	0.42***	0.44***	0.31***
	(0.15)	(0.10)	(0.08)	(0.14)	(0.08)	(0.08)
Observations	910	1,088	998	1,150	2,012	1,726
R-squared	48	48	48	24	42	36
Classification	**RMBL**	**RMBL**	**RMBL**	**RMBL**	**RMBL**	**RMBL**

Note: Subject level fixed effects robust estimator fits for M regression models with cluster-robust standard error for panels nested within subject level. PE stands for prediction error, i.e., PE = $p_t - p_t^*$. *** p<0.01, ** p<0.05, * p<0.1.

Estimated Continuous Learning Speed: M-estimator, Split Sample

Table A.15: $\frac{d\Delta G_{i,t+1,t}}{dR_{i,t,t-1}}$ in Positive Feedback Market: Split Sample

Study and Description	LtFE in Positive and Negative Feedback Market / Bao et al. (2012), JEDC, ***Positive Feedback Markets***			Theory of Mind (ToM) / Bao et al (2024), JEBO				LtFE vs. LtOE Positive / Bao et al. (2017), EJ		Speculator vs. Supplier in Housing Market / Bao and Hommes (2019), JEDC		
Treatment	REE = 56, $1 \leq t \leq 20$	REE = 41, $21 \leq t \leq 43$	REE = 62, $44 \leq t \leq 65$	High-ToM	Medium High ToM	Medium Low ToM	Low-ToM	LtFE	LtFE+LtOE Both	No Supplier	Low PES	High PES
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: Error Smaller than Subject-Level Median (Small Error = 1)												
Positively Correlated PE	1.35***	3.84***	4.28***	5.27***	7.29***	7.23***	5.65***	1.27***	2.03***	3.86***	3.76***	3.59***
	(0.44)	(1.30)	(1.43)	(0.85)	(0.93)	(0.89)	(0.69)	(0.22)	(0.50)	(0.77)	(0.28)	(0.25)
Observations	459	502	490	2,290	2,281	2,318	2,275	1,174	1,173	583	1,069	1,283
R-squared	48	48	48	96	96	96	96	48	48	24	45	54
Panel B: Error Larger than or Equal to Subject-Level Median (Small Error = 0)												
Positively Correlated PE	1.88***	1.38***	4.66***	3.22***	4.81***	4.28***	3.34***	0.97***	1.94***	4.46***	2.75***	3.46***
	(0.28)	(0.48)	(0.74)	(0.27)	(0.40)	(0.31)	(0.25)	(0.10)	(0.22)	(0.65)	(0.21)	(0.17)
Observations	435	575	504	2,308	2,309	2,276	2,315	1,109	1,129	569	1,091	1,303
R-squared	48	48	48	96	96	96	96	48	48	24	45	54

Note: Subject level fixed effects robust estimator fits for M regression models with cluster-robust standard error for panels nested within subject level. PE stands for prediction error, i.e., PE $= p_t - p_t^*$. *** p<0.01, ** p<0.05, * p<0.1.

Table A.16: $\frac{d\Delta G_{i,t+1,t}}{dR_{i,t,t-1}}$ in Negative Market: Split Sample

Study and Description	**LtFE in Positive and Negative Feedback Market / Bao et al. (2012), JEDC, *Negative Feedback Markets***			**LtFE vs. LtOE Negative / Bao et al. (2013), EER**		
Treatment	REE = 56, $1 \leq t \leq 20$	REE = 41, $21 \leq t \leq 43$	REE = 62, $44 \leq t \leq 65$	LtFE	LtFE+LtOE Both	LtFE+LtOE Either
	(13)	(14)	(15)	(16)	(17)	(18)
Panel A: Error Smaller than Subject-Level Median (Small Error = 1)						
Positively Correlated PE	0.47**	0.29*	0.34**	-0.24	-0.14	0.14
	(0.20)	(0.16)	(0.15)	(0.18)	(0.15)	(0.15)
Observations	474	509	486	590	1,023	872
R-squared	48	48	48	24	42	36
Panel B: Error Larger than or Equal to Subject-Level Median (Small Error = 0)						
Positively Correlated PE	1.35***	0.82***	1.02***	0.73***	0.59***	0.69***
	(0.17)	(0.11)	(0.11)	(0.17)	(0.09)	(0.11)
Observations	436	579	512	560	989	854
R-squared	48	48	48	24	42	36

Note: Subject level fixed effects robust estimator fits for M regression models with cluster-robust standard error for panels nested within subject level. PE stands for prediction error, i.e., PE = $p_t - p_t^*$. *** p<0.01, ** p<0.05, * p<0.1.

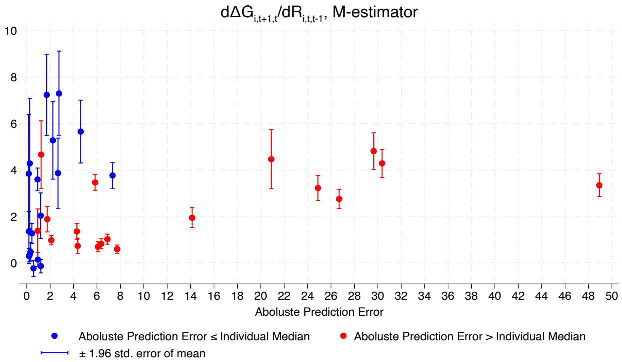


Figure A.3: M-estimator: coefficients of $d\Delta G_{i,t+1,t}/dR_{i,t,t-1}$ with respect to absolute prediction error in the sample in 18 experiments, separated by its absolute prediction error with respect to individual median.

finding

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